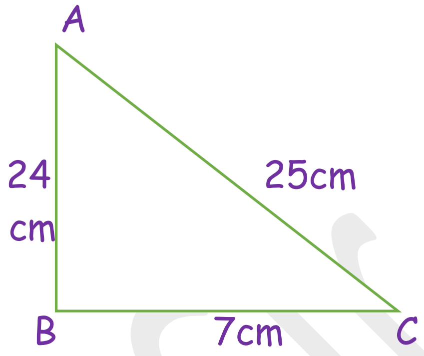
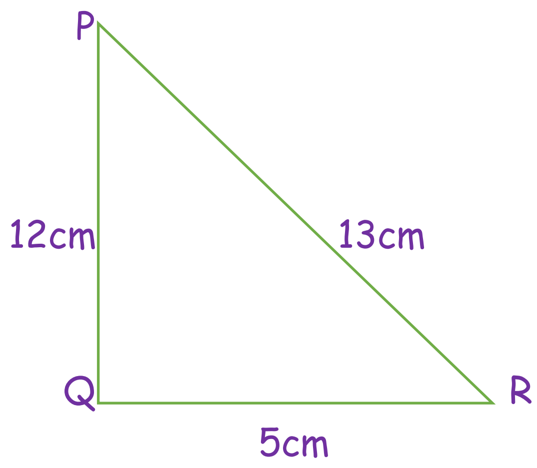
Exercise 8.1

1. In Δ ABC, right-angled at B, AB = 24 cm, BC = 7 cm. Determine:
(i) sin A, cos A
(ii) sin C, cos C



Sol. In the given triangle ABC, ∠B = 90
And AB = 24 cm and BC = 7 cm
By applying Pythagoras theorem, we get
$AC^2 = AB^2 + BC^2$
$AC^2 = 24^2 + 7^2$
$AC^2 = 576 + 49$
$AC^2 = 625^2$
$AC^2 = 25^2$
Therefore, AC = 25 cm
(i) sin A, cos A
$\text{Sin } A = \frac{\text{Opposite side}}{\text{Hypotenuse}} = \frac{BC}{AC} = \frac{7}{25}$
$\text{Cos } A = \frac{\text{Adjacent side}}{\text{Hypotenuse}} = \frac{AB}{AC} = \frac{24}{25}$
(ii) sin C, cos C
$\text{Sin } C = \frac{\text{Opposite side}}{\text{Hypotenuse}} = \frac{AB}{AC} = \frac{24}{25}$
$\text{Cos } C = \frac{\text{Adjacent side}}{\text{Hypotenuse}} = \frac{BC}{AC} = \frac{7}{25}$

2. In Fig., find tan P – cot R
Sol. In the given triangle PQR, the given triangle is right angled at Q and the given measures are:
PR = 13cm,
PQ = 12cm



According to Pythagorean theorem,
$PR^2 = QR^2 + PQ^2$
$13^2 = QR^2 + 12^2$
$169 = QR^2 + 144$
$\therefore QR^2 = 169 - 144$
$QR^2 = 25$
$QR = \sqrt{25} = 5$
Therefore, the side QR = 5 cm

$\tan(P) = \frac{\text{Opposite side}}{\text{Adjacent side}} = \frac{QR}{PQ} = \frac{5}{12}$

$\cot(R) = \frac{\text{Adjacent side}}{\text{Opposite side}} = \frac{QR}{PQ} = \frac{5}{12}$

$\therefore \tan(P) - \cot(R) = \frac{5}{12} - \frac{5}{12}$

$= 0$

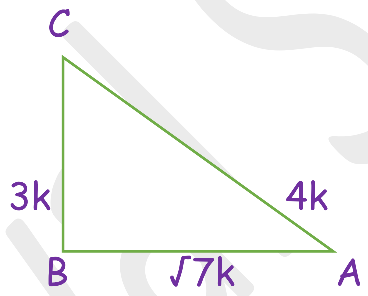
Therefore, tan P – cot P = 0 Ans.

3. If $\sin A = \frac{3}{4}$, Calculate cos A and tan A.

Sol. Given: $\sin A = \frac{3}{4}$

$\frac{\text{Opposite side}}{\text{Hypotenuse}} = \frac{3}{4}$

$\frac{BC}{AC} = \frac{3}{4}$



Let BC = 3k and AC = 4k, where k is a positive real number.

By applying Pythagoras theorem, we get

$AB^2 + BC^2 = AC^2$

$AB^2 + (3k)^2 = (4k)^2$

$AB^2 = 16k^2 - 9k^2$

$AB^2 = 7k^2$

$\therefore AB = \sqrt{7}k$

$\cos A = \frac{\text{Adjacent side}}{\text{Hypotenuse}} = \frac{AB}{AC} = \frac{\sqrt{7}k}{4k} = \frac{\sqrt{7}}{4}$

$\tan A = \frac{\text{Opposite side}}{\text{Adjacent side}} = \frac{BC}{AB} = \frac{3k}{\sqrt{7}k} = \frac{3}{\sqrt{7}}$

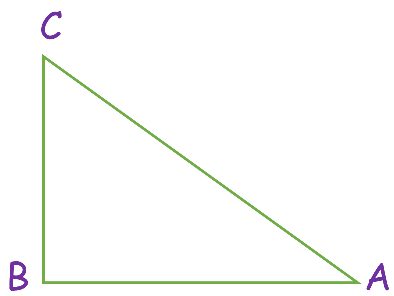
4. Given 15 cot A = 8, find sin A and sec A.

Sol. Given: 15 cot A = 8

So, $\cot A = \frac{8}{15}$

$\frac{\text{Adjacent side}}{\text{Opposite side}} = \frac{8}{15}$

$\frac{AB}{BC} = \frac{8}{15}$



Let AB = 8k and BC = 15k, where, k is a positive real number.

$AC^2 = AB^2 + BC^2$

$AC^2 = (8k)^2 + (15k)^2$

$AC^2 = 64k^2 + 225k^2$

$AC^2 = 289k^2$

$\therefore AC = 17k$

$$\text{Sin}(A) = \frac{\text{Opposite side}}{\text{Hypotenuse}} = \frac{BC}{AC} = \frac{15k}{17k} = \frac{15}{17}$$

$$\text{Sec}(A) = \frac{\text{Hypotenuse}}{\text{Adjacent side}} = \frac{AC}{AB} = \frac{17k}{8k} = \frac{17}{8}$$

5. Given $\sec\theta = \frac{13}{12}$ calculate all other trigonometric ratios

Sol. $\sec\theta = \frac{13}{12}$

$$\frac{\text{Hypotenuse}}{\text{Adjacent side}} = \frac{13}{12}$$

$$\frac{AC}{AB} = \frac{13}{12}$$

Let AC = 13k and AB = 12k, where, k is a positive real number.

By using Pythagoras theorem, we get

$AB^2 + BC^2 = AC^2$

$(12k)^2 + BC^2 = (13k)^2$

$144k^2 + BC^2 = 169k^2$

$BC^2 = 169k^2 - 144k^2$

$BC^2 = 25k^2$

$\therefore BC = \sqrt{25}\,k$

$\therefore BC = 5\,k$

$$\text{Sin}\,\theta = \frac{\text{Opposite side}}{\text{Hypotenuse}} = \frac{BC}{AC} = \frac{5}{13}$$

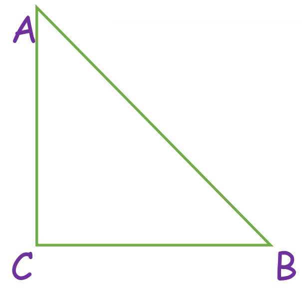
$$\text{Cos}\,\theta = \frac{\text{Adjacent side}}{\text{Hypotenuse}} = \frac{AB}{AC} = \frac{12}{13}$$

$$\tan\theta = \frac{\text{Opposite side}}{\text{Adjacent side}} = \frac{BC}{AB} = \frac{5}{12}$$

$$\cot\theta = \frac{\text{Adjacent side}}{\text{Opposite side}} = \frac{AB}{BC} = \frac{12}{5}$$

$$\text{Cosec}\,\theta = \frac{\text{Hypotenuse}}{\text{Opposite side}} = \frac{AC}{BC} = \frac{13}{5}$$

6. If ∠A and ∠B are acute angles such that cos A = cos B, then show that ∠ A = ∠ B.



Sol. In triangle ABC, ∠ C is 90^0 such that

cos A = cos B

$\frac{AC}{AB} = \frac{BC}{AB}$

AC = BC

∠ A = ∠ B (angles opposite to equal angles are equal)

7. If cot θ = $\frac{7}{8}$, evaluate: (i) $\frac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)}$ (ii) $\cot^2\theta$

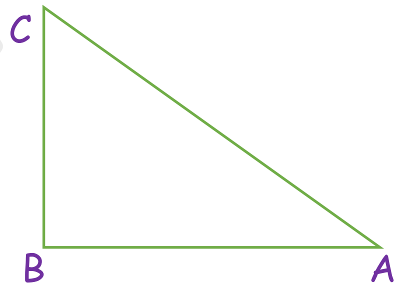
Sol. Let us assume a △ABC in which ∠B = 90° and ∠A = θ

cot θ = $\frac{7}{8}$

$\frac{\text{Adjacent side}}{\text{Opposite side}} = \frac{AB}{BC}$

Let AB = 7k and BC = 8k, where k is a positive real number

According to Pythagoras theorem in △ABC we get.



$AC^2 = AB^2 + BC^2$

$AC^2 = (7k)^2 + (8k)^2$

$AC^2 = 49k^2 + 64k^2$

$AC^2 = 113k^2$

∴ AC = $\sqrt{113}$ k

$\sin\theta = \frac{\text{Opposite side}}{\text{Hypotenuse}} = \frac{BC}{AC} = \frac{8K}{\sqrt{113}\,k} = \frac{8}{\sqrt{113}}$

$\cos\theta = \frac{\text{Adjacent side}}{\text{Hypotenuse}} = \frac{AB}{AC} = \frac{7K}{\sqrt{113}\,k} = \frac{7}{\sqrt{113}}$

$$\frac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)} = \frac{(1-\sin^2\theta)}{(1-\cos^2\theta)}$$

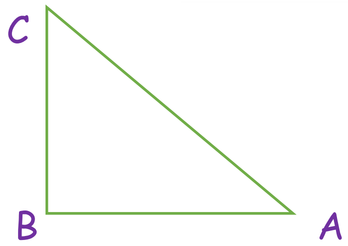
$$= \frac{(1-\frac{64}{113})}{(1-\frac{49}{113})}$$

$$= \frac{(\frac{113-64}{113})}{(\frac{113-49}{113})} = \frac{49}{64}$$ Ans.

(ii) $\cot^2\theta = \left(\frac{AB}{BC}\right)^2 = \left(\frac{7k}{8k}\right)^2 = \left(\frac{7}{8}\right)^2 = \frac{49}{64}$ Ans.

8. If $3\cot A = 4$, check whether $\frac{(1-\tan^2 A)}{(1+\tan^2 A)} = \cos^2 A - \sin^2 A$ or not.

Sol. Let $\triangle ABC$ in which $\angle B = 90°$



$3\cot A = 4$

$\cot A = \frac{4}{3}$

$\frac{AB}{BC} = \frac{4}{3}$

Let $AB = 4k$ an $BC = 3k$, where k is a positive real number.

Using Pythagorean theorem,

$AC^2 = AB^2 + BC^2$

$(AC)^2 = (4k)^2 + (3k)^2$

$(AC)^2 = 16k^2 + 9k^2$

$AC^2 = 25k^2$

$AC = 5k$

tan(A) = $\frac{\text{Opposite side}}{\text{Adjacent side}} = \frac{BC}{AB} = \frac{3k}{4k} = \frac{3}{4}$

Sin (A) = $\frac{\text{Opposite side}}{\text{Hypotenuse}} = \frac{BC}{AC} = \frac{3k}{5k} = \frac{3}{5}$

Cos (A) = $\frac{\text{Adjacent side}}{\text{Hypotenuse}} = \frac{AB}{AC} = \frac{4k}{5k} = \frac{4}{5}$

L. H. S. = $\frac{(1-\tan^2 A)}{(1+\tan^2 A)}$

$$= \frac{1-\left(\frac{3}{4}\right)^2}{1+\left(\frac{3}{4}\right)^2} = \frac{1-\frac{9}{16}}{1+\frac{9}{16}} = \frac{\frac{7}{16}}{\frac{25}{16}} = \frac{\frac{7}{16}}{\frac{25}{16}} = \frac{7}{25}$$

R. H. S. = $\cos^2 A - \sin^2 A$

$$= \left(\frac{4}{5}\right)^2 - \left(\frac{3}{5}\right)^2 = \frac{16}{25} - \frac{9}{25} = \frac{7}{25}$$

R.H.S. =L.H.S.

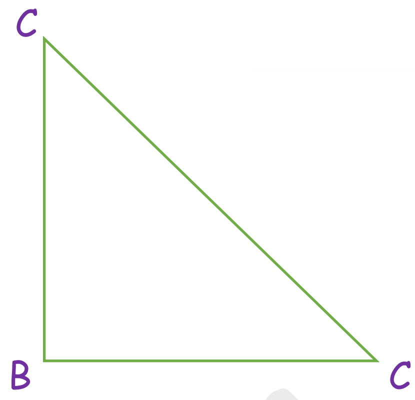
Hence, $\frac{(1-\tan^2 A)}{(1+\tan^2 A)} = \cos^2 A - \sin^2 A$ are equal.

9. In triangle ABC, right-angled at B, if $\tan A = \frac{1}{\sqrt{3}}$, find the value of:

(i) $\sin A \cos C + \cos A \sin C$

(ii) $\cos A \cos C - \sin A \sin C$

Sol. In ΔABC, $\angle B = 90°$



$\tan A = \frac{1}{\sqrt{3}}$

$\frac{BC}{AB} = \frac{1}{\sqrt{3}}$

Let $BC = 1k$ and $AB = \sqrt{3}\,k$,

where k is a positive real number.

By using Pythagoras theorem, we get:

$AC^2 = AB^2 + BC^2$

$AC^2 = (\sqrt{3}k)^2 + k^2$

$AC^2 = 3k^2 + k^2$

$AC^2 = 4k^2$

$AC = 2k$

(i) $\sin A = \frac{BC}{AC} = \frac{1}{2}$ and $\cos A = \frac{AB}{AC} = \frac{\sqrt{3}}{2}$

(ii) $\sin C = \frac{AB}{AC} = \frac{\sqrt{3}}{2}$ and $\cos C = \frac{BC}{AC} = \frac{1}{2}$

(i) $\sin A \cos C + \cos A \sin C = \left(\frac{1}{2}\right) \times \left(\frac{1}{2}\right) + \left(\frac{\sqrt{3}}{2}\right) \times \left(\frac{\sqrt{3}}{2}\right)$

$= \frac{1}{4} + \frac{3}{4}$

$= 1$

(ii) $\cos A \cos C - \sin A \sin C = \left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{2}\right) - \left(\frac{1}{2}\right)\left(\frac{\sqrt{3}}{2}\right)$

$= \frac{\sqrt{3}}{4} - \frac{\sqrt{3}}{4}$

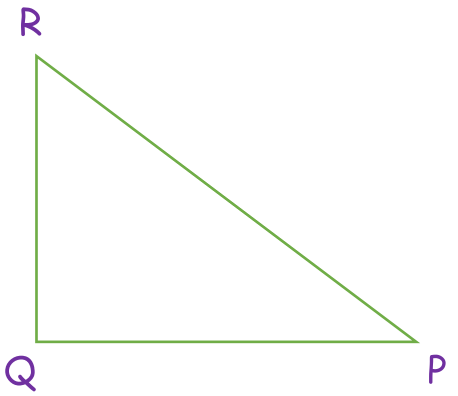
$= 0$ Ans.

10. In ΔPQR, right-angled at Q, $PR + QR = 25$ cm and $PQ = 5$ cm. Determine the values of $\sin P$, $\cos P$ and $\tan P$

Sol. In ΔPQR, $\angle Q = 90°$,

$PR + QR = 25$ cm and $PQ = 5$ cm

Let $QR = x$



$PR = 25 - QR$

$PR = 25 - x$

Using Pythagorean Theorem,

$PQ^2 + QR^2 = PR^2$

$\Rightarrow 5^2 + x^2 = (25 - x)^2$

$\Rightarrow 25 + x^2 = 25^2 + x^2 - 50x$

$\Rightarrow 25 + x^2 = 625 + x^2 - 50x$

$\Rightarrow x^2 - x^2 + 50x = 625 - 25$

$\Rightarrow 50x = 600$

$\Rightarrow x = \dfrac{600}{50}$

$\therefore\ x = 12$

$\therefore$ QR $= 12$ cm

and PR $= 25 - \text{QR}$

PR $= 25 - 12$

$\therefore$ PR $= 13$ cm

sin P = $\dfrac{\text{Opposite side}}{\text{Hypotenuse}} = \dfrac{QR}{PR} = \dfrac{12}{13}$

cos P = $\dfrac{\text{Opposite side}}{\text{Hypotenuse}} = \dfrac{PQ}{PR} = \dfrac{5}{13}$

tan P = $\dfrac{\text{Opposite side}}{\text{Hypotenuse}} = \dfrac{QR}{PQ} = \dfrac{12}{5}$

$\therefore$the values of $\sin P = \dfrac{12}{13}$,

$\cos P = \dfrac{5}{13}$

and $\tan P = \dfrac{12}{5}$ Ans.

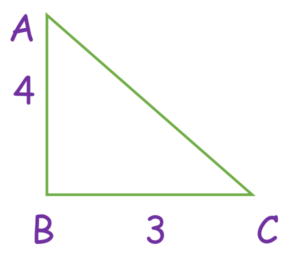
11. State whether the following are true or false. Justify your answer.

(i) The value of tan A is always less than 1.

Sol. False

Here tan C = $\frac{AB}{BC} = \frac{4}{3}$

$\frac{4}{3}$ = 1.333….> 1



(ii) sec A = 12/5 for some value of angle A.

Sol. true

Justification: In triangle ABC

sec A = $\dfrac{\text{Hypotenuse}}{\text{Adjacent side}} = \frac{12}{5}$

Here hypotenuse > adjacent side

(iii)cos A is the abbreviation used for the cosecant of angle A.

Sol. False

Justification: cos A is the abbreviation used for the cosine of angle A while abbreviation used for cosecant of angle A is cosec A.

(iv) cot A is the product of cot and A.

Sol. False

Justification: cot A is not the product of cot and A.

cot A is the abbreviation used for the cotangent of angle A.

(v) sin θ = 4/3 for some angle θ.

Sol. False

Justification: Sin θ = $\frac{\text{Opposite side}}{\text{Hypotenuse}}$

We know that Hypotenuse is the longest side in a right-angled triangle.

∴ Sin θ will always less than 1 and it can never be $\frac{4}{3}$ for any value of θ.

Exercise 8.2

1. Evaluate the following:

(i) $\sin 60° \cos 30° + \sin 30° \cos 60°$

Sol. $\sin 60° \cos 30° + \sin 30° \cos 60° = \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} + \frac{1}{2} \times \frac{1}{2}$

$= \frac{3}{4} + \frac{1}{4}$

$= \frac{4}{4}$

$= 1$

∴ $\sin 60° \cos 30° + \sin 30° \cos 60° = 1$ Ans.

(ii) $2\tan^2 45° + \cos^2 30° - \sin^2 60°$

Sol. $2\tan^2 45° + \cos^2 30° - \sin^2 60° = 2(1)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 - \left(\frac{\sqrt{3}}{2}\right)^2$

$= 2 + \frac{3}{4} - \frac{3}{4}$

$= 2$

∴ $2tan^2 45° + cos^2 30° - sin^2 60° = 2$ Ans.

(iii) $\frac{\cos 45°}{\sec 30° + \text{cosec} 30°}$

Sol. $\frac{cos 45°}{\sec 30° + cosec 30°} = \frac{\frac{1}{\sqrt{2}}}{\frac{2}{\sqrt{3}} + 2}$

$$= \frac{\frac{1}{\sqrt{2}}}{\frac{2+2\sqrt{3}}{\sqrt{3}}}$$

$$= \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2(1+\sqrt{3})}$$

$$= \frac{\sqrt{2}}{2} \times \frac{\sqrt{3}}{2(\sqrt{3}+1)}$$

$$= \frac{\sqrt{6}}{4(\sqrt{3}+1)} \times \frac{\sqrt{3}-1}{\sqrt{3}-1}$$

$$= \frac{\sqrt{18}-1}{4(3-1)}$$

$$= \frac{3\sqrt{2}-\sqrt{6}}{4\times 2}$$

$$= \frac{3\sqrt{2}-\sqrt{6}}{8}$$

$$\therefore \frac{cos45°}{\sec 30° + cosec30°} = \frac{3\sqrt{2}-\sqrt{6}}{8}$$ Ans.

(iv) $\frac{\sin30°+\tan45°-cosec60°}{\sec30°+\cos60°+\cot45°}$

Sol. $$\frac{sin30°+tan45°-cosec60°}{sec30°+cos60°+cot45°} = \frac{\frac{1}{2}+1-\frac{2}{\sqrt{3}}}{\frac{2}{\sqrt{3}}+\frac{1}{2}+1}$$

$$= \frac{\frac{\sqrt{3}+2\sqrt{3}-4}{2\sqrt{3}}}{\frac{\sqrt{3}+2\sqrt{3}+4}{2\sqrt{3}}}$$

$$= \frac{3\sqrt{3}-4}{3\sqrt{3}+4} \times \frac{3\sqrt{3}-4}{3\sqrt{3}-4}$$

$$= \frac{(3\sqrt{3}-4)^2}{(3\sqrt{3})^2-4^2}$$

$$= \frac{(3\sqrt{3})^2+(4)^2-2.3\sqrt{3}.4}{27-16}$$

$$= \frac{27+16-24\sqrt{3}}{27-16}$$

$$= \frac{43-24\sqrt{3}}{11}$$

$$\therefore \frac{sin30°+tan45°-cosec60°}{sec30°+cos60°+cot45°} = \frac{1}{11}(43-24\sqrt{3})$$ Ans.

(v) $\frac{5\cos^2 60° + 4\sec^2 30° - \tan^2 45°}{\sin^2 30° + \cos^2 30°}$

Sol. $\frac{(5\cos^2 60° + 4\sec^2 30° - \tan^2 45°)}{(\sin^2 30° + \cos^2 30°)} = \frac{5\left(\frac{1}{2}\right)^2 + 4\left(\frac{2}{\sqrt{3}}\right)^2 - 1^2}{\left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2}$

$$= \frac{\frac{5}{4} + \frac{16}{3} - 1}{\frac{1}{4} + \frac{3}{4}}$$

$$= \frac{\frac{15 + 64 - 12}{4}}{\frac{1+3}{4}}$$

$$= \frac{\frac{79-12}{12}}{\frac{4}{4}}$$

$$= \frac{67}{12} \times \frac{4}{4}$$

$$= \frac{67}{12}$$

$\therefore \frac{5\cos^2 60° + 4\sec^2 30° - \tan^2 45°}{\sin^2 30° + \cos^2 30°} = \frac{67}{12}$ Ans.

2. Choose the correct option and justify your choice:

(i) $\frac{2\tan 30°}{1 + \tan^2 30°} =$

(A) $\sin 60°$ (B) $\cos 60°$ (C) $\tan 60°$ (D) $\sin 30°$

Sol. $\frac{2\tan 30°}{1 + \tan^2 30°} = \frac{2 \cdot \frac{1}{\sqrt{3}}}{1 + \left(\frac{1}{\sqrt{3}}\right)^2}$

$$= \frac{2 \cdot \frac{1}{\sqrt{3}}}{1 + \left(\frac{1}{\sqrt{3}}\right)^2}$$

$$= \frac{2 \cdot \frac{1}{\sqrt{3}}}{1 + \frac{1}{3}} = \frac{\frac{2}{\sqrt{3}}}{\frac{4}{3}} = \frac{2}{\sqrt{3}} \times \frac{3}{4} = \frac{\sqrt{3}}{2} = \sin 60°$$

∴ Option A is correct.

(ii) $\frac{1 - \tan^2 45°}{1 + \tan^2 45°} =$

(A) $\tan 90°$ (B) 1 (C) $\sin 45°$ (D) 0

Sol. $\frac{1-tan^2 45°}{1+tan^2 45°} = \frac{1-1^2}{1+1^2} = \frac{0}{2} = 0$

∴ Option D is correct.

(iii) $sin2A = 2sinA$ is true when A =

(A) $0°$ (B) $30°$ (C) $45°$ (D) $60°$

Sol. $sin2A = 2sinA$

For A = $0°$

$sin2 . 0° = 2 \sin 0°$

$sin\ 0° = 2 \sin 0°$

0 = 2 . 0

0 = 0

Option A is correct.

(iv) $\frac{2tan30°}{1 - tan^2 30°} =$

(A) $cos60°$ (B) $sin60°$ (C) $tan60°$ (D) $sin30°$

Sol. $\frac{2tan30°}{1 + tan^2 30°} = \frac{2 . \frac{1}{\sqrt{3}}}{1-\left(\frac{1}{\sqrt{3}}\right)^2} = \frac{2 . \frac{1}{\sqrt{3}}}{1-\left(\frac{1}{\sqrt{3}}\right)^2} = \frac{2 . \frac{1}{\sqrt{3}}}{1-\frac{1}{3}}$

$= \frac{\frac{2}{\sqrt{3}}}{\frac{2}{3}}$

$= \frac{2}{\sqrt{3}} \times \frac{3}{2}$

$= \sqrt{3}$

= tan 60°

∴ Option C is correct.

3. If tan (A + B) = √3 and tan (A – B) = 1/√3 ,0° < A + B ≤ 90°; A > B, find A and B.

Sol. tan (A + B) = √3

⇒tan (A + B) = tan 60° [∵ tan 60° = √3]

⇒(A + B) = 60° … (i)

And tan (A – B) = $\frac{1}{\sqrt{3}}$

⇒tan (A – B) = tan 30° [∵ tan 30° = $\frac{1}{\sqrt{3}}$]

⇒ (A – B) = 30° … equation (ii)

Now adding (i) and (ii), we get

A + B + A – B = 60° + 30°

2A = 90°

∴ A= 45°

Now, put the value of A in equation (i)

45° + B = 60°

B = 60° – 45°

B = 15°

∴ A = 45° and B = 15° Ans.

4. State whether the following are true or false. Justify your answer.

(i) sin (A + B) = sin A + sin B.

Sol. (i) False.

Justification:

Let us take A = 30° and B = 60°, then

sin (A + B) = sin (30° + 60°) = sin 90° = 1 and,

sin A + sin B = sin 30° + sin 60°

$= \frac{1}{2} + \frac{\sqrt{3}}{2}$

$= \frac{1+\sqrt{3}}{2}$

Here sin (A + B) ≠ sin A + sin B.

So, the given statement is false.

(ii) The value of sin θ increases as θ increases.

Sol. True.

Justification: the values of sin θ are:

θ→	0°	30°	45°	60°	90°
sin θ	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1

So, the value of sin θ increases as θ increases. Hence, the statement is true

(iii) The value of cos θ increases as θ increases.

Sol. False.

Justification:

the values of sin θ are:

θ→	0°	30°	45°	60°	90°
Cos θ	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	1

So, the value of cos θ decreases as θ increases. Hence, the statement is false

(iv) sin θ = cos θ for all values of θ.

Sol. We know

sin 90° =1 and cos 90° = 0

Here sin 90° ≠ cos 90°

∴ the above statement is false.

(v) cot A is not defined for A = 0°.

Sol. We know cot A = $\frac{\cos A}{\sin A} = \frac{\cos 0°}{\sin 0°} = \frac{1}{0}$ = undefined

∴ the above statement is true

Exercise 8.3

1. Express the trigonometric ratios sin A, sec A and tan A in terms of cot A.

Sol. sin A $= \frac{1}{Cosec\ A}$

$= \frac{1}{\sqrt{cosec^2 A}}$

$= \frac{1}{\sqrt{1+ cot^2 A}}$ [$cosec^2 A = 1 + cot^2 A$]

sec A$= \sqrt{sec^2 A}$

$= \sqrt{1 + tan^2 A}$

$= \sqrt{1 + \frac{1}{cot^2 A}}$

$= \sqrt{\frac{cot^2 A + 1}{cot^2 A}}$

$= \frac{\sqrt{cot^2 A + 1}}{\cot A}$

tan A $= \frac{1}{\cot A}$ Ans.

2. Write all the other trigonometric ratios of ∠A in terms of sec A.

Sol. sin A$= \sqrt{sin^2 A}$

$= \sqrt{1 - cos^2 A}$

$= \sqrt{1 - \frac{1}{sec^2 A}}$

$= \sqrt{\frac{sec^2 A - 1}{sec^2 A}}$

$$= \frac{\sqrt{sec^2 A + 1}}{\sec A}$$

$\cos A = \frac{1}{\sec A}$

$\tan A = \sqrt{tan^2 A}$

$= \sqrt{sec^2 A - 1}$

$\text{Cot } A = \frac{1}{\tan A}$

$= \frac{1}{\sqrt{tan^2 A}}$

$= \frac{1}{\sqrt{sec^2 A - 1}}$

$\text{cosec } A = \frac{1}{\sin A}$

$= \frac{1}{\frac{\sqrt{sec^2 A + 1}}{\sec A}}$ [from (i) $\sin A = \frac{\sqrt{sec^2 A + 1}}{\sec A}$]

$= \frac{\sec A}{\sqrt{\sec^2 A + 1}}$

3. Choose the correct option. Justify your choice.

(i) $9 \sec^2 A - 9 \tan^2 A =$

(A) 1 (B) 9 (C) 8 (D) 0

Sol. $9 \sec^2 A - 9 \tan^2 A = 9 (\sec^2 A - \tan^2 A)$

$= 9 \times 1$ ($\because \sec^2 A - \tan^2 A = 1$)

$= 9$

Option (B) is correct.

(ii) $(1 + \tan\theta + \sec\theta)(1 + \cot\theta - \text{cosec}\,\theta)$

(A) 0 (B) 1 (C) 2 (D) - 1

Sol. $(1 + \tan\theta + \sec\theta)(1 + \cot\theta - \text{cosec}\,\theta)$

$= \left(1 + \frac{\sin\theta}{\cos\theta} + \frac{1}{\cos\theta}\right)\left(1 + \frac{\cos\theta}{\sin\theta} - \frac{1}{\sin\theta}\right)$

$= \left(\frac{\cos\theta + \sin\theta + 1}{\cos\theta}\right)\left(\frac{\sin\theta + \cos\theta - 1}{\sin\theta}\right)$

$= \left(\frac{sin^2\theta + cos^2\theta + 2\cos\theta.\ \sin\theta - 1}{\cos\theta.\ \sin\theta}\right)$

= $\left(\frac{1+2\cos\theta.\ \sin\theta-1}{\cos\theta.\ \sin\theta}\right)$

= $\left(\frac{2\cos\theta.\ \sin\theta}{\cos\theta.\ \sin\theta}\right)$

= 2

Option (C) is correct.

(iii) (sec A + tan A) (1 – sin A) =

(A) sec A (B) sin A (C) cosec A (D) cos A

Sol. (sec A + tan A) (1 – sin A)

= $\left(\frac{1}{\cos A} + \frac{\sin A}{\cos A}\right)(1 - \sin A)$

= $\left(\frac{1+\sin A}{\cos A}\right)(1 - \sin A)$

= $\frac{(1+\sin A)(1-\sin A)}{\cos A}$

= $\frac{(1-sin^2 A)}{\cos A}$

= $\frac{cos^2 A}{\cos A}$

= cos A

Option (D) is correct.

(iv) $\frac{1+tan^2 A}{1+cot^2 A}$ =

(A) sec^2 A (B) -1 (C) $cot^2 A$ (D) $tan^2 A$

Sol. $\frac{1+tan^2 A}{1+cot^2 A} = \frac{sec^2 A}{cosec^2 A}$

$= \frac{sin^2 A}{cos^2 A}$

$= tan^2 A$

Option (D) is correct

4. Prove the following identities, where the angles involved are acute angles for which the expressions are defined.

(i) $(\operatorname{cosec}\theta - \cot\theta)^2 = \frac{1-\cos\theta}{1+\cos\theta}$

Sol. R.H.S. $= \frac{1-\cos\theta}{1+\cos\theta}$

$= \frac{1-\cos\theta}{1+\cos\theta} \times \frac{1-\cos\theta}{1-\cos\theta}$

$= \frac{(1-\cos\theta)^2}{1-\cos^2\theta}$

$= \frac{(1-\cos\theta)^2}{\sin^2\theta}$

$= \left(\frac{1-\cos\theta}{\sin\theta}\right)^2$

$= \left(\frac{1}{\sin\theta} - \frac{\cos\theta}{\sin\theta}\right)^2$

$= (\operatorname{cosec}\theta - \cot\theta)^2$

= L.H.S

Hence $(\operatorname{cosec}\theta - \cot\theta)^2 = \frac{1-\cos\theta}{1+\cos\theta}$ proved.

(ii) $\frac{\cos A}{1+\sin A} + \frac{1+\sin A}{\cos A} = 2\sec A$

Sol. L.H.S. $= \frac{\cos A}{1+\sin A} + \frac{1+\sin A}{\cos A}$

$= \frac{\cos^2 A + (1+\sin A)^2}{(1+\sin A)\cos A}$

$= \frac{\cos^2 A + 1+\sin^2 A+2\sin A}{(1+\sin A)\cos A}$ $[(a + b)^2 = a^2 + b^2 + 2ab]$

$= \frac{\cos^2 A + \sin^2 A + 1+2\sin A}{(1+\sin A)\cos A}$

$= \frac{1 + 1+2\sin A}{(1+\sin A)\cos A}$

$= \frac{2+2\sin A}{(1+\sin A)\cos A}$

$= \frac{2(1+\sin A)}{(1+\sin A)\cos A}$

$= \frac{2}{\cos A}$

$= 2 \sec A$

= R.H.S.

Hence $\frac{\cos A}{1+\sin A} + \frac{1+\sin A}{\cos A} = 2 \sec A$ proved.

(iii) $\frac{\tan\theta}{1-\cot\theta} + \frac{\cot\theta}{1-\tan\theta} = 1 + \sec\theta \operatorname{cosec}\theta$

Sol. L.H.S. $= \frac{\tan\theta}{1-\cot\theta} + \frac{\cot\theta}{1-\tan\theta}$

$= \frac{\frac{\sin\theta}{\cos\theta}}{1-\frac{\cos\theta}{\sin\theta}} + \frac{\frac{\cos\theta}{\sin\theta}}{1-\frac{\sin\theta}{\cos\theta}}$

$= \frac{\frac{\sin\theta}{\cos\theta}}{\frac{\sin\theta-\cos\theta}{\sin\theta}} + \frac{\frac{\cos\theta}{\sin\theta}}{\frac{\cos\theta-\sin\theta}{\cos\theta}}$

$= \frac{\frac{\sin\theta}{\cos\theta}}{\frac{\sin\theta-\cos\theta}{\sin\theta}} + \frac{\frac{\cos\theta}{\sin\theta}}{\frac{-(-\cos\theta+\sin\theta)}{\cos\theta}}$

$= \frac{\frac{\sin\theta}{\cos\theta}}{\frac{\sin\theta-\cos\theta}{\sin\theta}} - \frac{\frac{\cos\theta}{\sin\theta}}{\frac{(\sin\theta-\cos\theta)}{\cos\theta}}$

$= \frac{\sin\theta}{\cos\theta}\frac{\sin\theta}{(\sin\theta-\cos\theta)} - \frac{\cos\theta}{\sin\theta}\frac{\cos\theta}{(\sin\theta-\cos\theta)}$

$= \frac{\sin^2\theta}{\cos\theta(\sin\theta-\cos\theta)} - \frac{\cos^2\theta}{\sin\theta(\sin\theta-\cos\theta)}$

$= \frac{\sin^3\theta-\cos^3\theta}{\cos\theta\sin\theta(\sin\theta-\cos\theta)}$

$= \frac{(\sin\theta-\cos\theta)(\sin^2\theta+\cos^2\theta+\cos\theta\sin\theta)}{\cos\theta\sin\theta(\sin\theta-\cos\theta)}$

$[a^3 - b^3 = (a-b)(a^2 + ab + b^2)]$

$= \frac{(1 + \cos\theta \sin\theta)}{\cos\theta \sin\theta}$

$= \frac{1}{\cos\theta \sin\theta} + \frac{\cos\theta \sin\theta}{\cos\theta \sin\theta}$

$= \sec\theta \operatorname{cosec}\theta + 1$

Hence $\frac{\tan\theta}{1-\cot\theta} + \frac{\cot\theta}{1-\tan\theta} = 1 + \sec\theta \operatorname{cosec}\theta$ proved.

(iv) $\frac{1 + \sec A}{\sec A} = \frac{\sin^2 A}{1-\cos A}$

Sol. L.H.S. $= \frac{1 + \sec A}{\sec A}$

$= \frac{1 + \frac{1}{\cos A}}{\frac{1}{\cos A}}$

$= \frac{\frac{\cos A+1}{\cos A}}{\frac{1}{\cos A}}$

$= \frac{\cos A+1}{\cos A} \times \frac{\cos A}{1}$

$= \cos A + 1$

$= (1 + \cos A) \times \frac{1-\cos A}{1-\cos A}$

$= \frac{1-\cos^2 A}{1-\cos A}$

$= \frac{\sin^2 A}{1-\cos A}$

Hence $\frac{1 + \sec A}{\sec A} = \frac{\sin^2 A}{1-\cos A}$ proved.

(v) $\frac{\cos A-\sin A+1}{\cos A+\sin A-1} = \operatorname{cosec} A + \cot A$, using the identity $\operatorname{cosec}^2 A = 1 + \cot^2 A$.

Sol. L.H.S. $= \frac{\cos A-\sin A+1}{\cos A+\sin A-1}$

Divide the numerator and denominator by sin A, we get

$= \dfrac{\cot A - 1 + \operatorname{cosec} A}{\cot A + 1 - \operatorname{cosec} A}$

$= \dfrac{\cot A - (cosec^2 A - cot^2 A) + \operatorname{cosec} A}{\cot A + 1 - \operatorname{cosec} A}$ (using $cosec^2 A - cot^2 A = 1$)

$= \dfrac{\cot A + \operatorname{cosec} A - (cosec^2 A - cot^2 A)}{\cot A + 1 - \operatorname{cosec} A}$

$= \dfrac{\cot A + \operatorname{cosec} A - (\operatorname{cosec} A + \cot A)(\operatorname{cosec} A - \cot A)}{\cot A + 1 - \operatorname{cosec} A}$

$= \dfrac{(\cot A + \operatorname{cosec} A)(1 - \operatorname{cosec} A + \cot A)}{\cot A + 1 - \operatorname{cosec} A}$

= cot A + cosec A

= R.H.S.

Hence $\dfrac{\cos A - \sin A + 1}{\cos A + \sin A - 1}$ = cosec A + cot A proved.

(vi) $\sqrt{\dfrac{1+\sin A}{1-\sin A}} = \sec A + \tan A$

Sol. L.H.S. = $\sqrt{\dfrac{1+\sin A}{1-\sin A}}$

$= \sqrt{\dfrac{1+\sin A}{1-\sin A} \times \dfrac{1+\sin A}{1+\sin A}}$

$= \sqrt{\dfrac{(1+\sin A)^2}{1-\sin^2 A}}$

$= \sqrt{\dfrac{(1+\sin A)^2}{\cos^2 A}}$

$= \dfrac{1+\sin A}{\cos A}$

$= \dfrac{1}{\cos A} + \dfrac{\sin A}{\cos A}$

$= \sec A + \tan A$

Hence $\sqrt{\dfrac{1+\sin A}{1-\sin A}} = \sec A + \tan A$ proved.

(vii) $\frac{\sin\theta - 2\sin^3\theta}{2\cos^3\theta - \cos\theta} = \tan\theta$

Sol. L.H.S. = $\frac{\sin\theta - 2\sin^3\theta}{2\cos^3\theta - \cos\theta}$

$= \frac{\sin\theta(1 - 2\sin^2\theta)}{\cos\theta(2\cos^2\theta - 1)}$

$= \frac{\sin\theta[1 - 2(1 - \cos^2\theta)]}{\cos\theta(2\cos^2\theta - 1)}$

$= \frac{\sin\theta[1 - 2 + 2\cos^2\theta)]}{\cos\theta(2\cos^2\theta - 1)}$

$= \frac{\sin\theta[2\cos^2\theta - 1]}{\cos\theta(2\cos^2\theta - 1)}$

$= \frac{\sin\theta}{\cos\theta}$

$= \tan\theta$

= R.H.S

Hence $\frac{\sin\theta - 2\sin^3\theta}{2\cos^3\theta - \cos\theta} = \tan\theta$ proved.

(viii) $(\sin A + \operatorname{cosec} A)^2 + (\cos A + \sec A)^2 = 7 + \tan^2 A + \cot^2 A$

Sol. L.H.S. = $(\sin A + \operatorname{cosec} A)^2 + (\cos A + \sec A)^2$

$= \sin^2 A + \operatorname{cosec}^2 A + 2\sin A \operatorname{cosec} A + \cos^2 A + \sec^2 A + 2\cos A \sec A$

$= \sin^2 A + \cos^2 A + 1 + \cot^2 A + 2\sin A \frac{1}{\sin A} + 1 + \tan^2 A + 2\cos A \frac{1}{\cos A}$

$= 1 + 1 + \cot^2 A + 2 + 1 + \tan^2 A + 2$

$= 7 + \tan^2 A + \cot^2 A$

= R.H.S.

Hence $(\sin A + \operatorname{cosec} A)^2 + (\cos A + \sec A)^2 = 7 + \tan^2 A + \cot^2 A$. proved.

(ix) $(\operatorname{cosec} A - \sin A)(\sec A - \cos A) = \frac{1}{\tan A + \cot A}$

Sol. L.H.S. = $(\operatorname{cosec} A - \sin A)(\sec A - \cos A)$

$= \left(\frac{1}{\sin A} - \sin A\right)\left(\frac{1}{\cos A} - \cos A\right)$

$= \left(\frac{1-\sin^2 A}{\sin A}\right)\left(\frac{1-\cos^2 A}{\cos A}\right)$

$= \frac{\cos^2 A}{\sin A} \times \frac{\sin^2 A}{\cos A}$

$= \cos A \sin A$

Now, R.H.S. $= \frac{1}{\tan A + \cot A}$

$= \frac{1}{\frac{\sin A}{\cos A} + \frac{\cos A}{\sin A}}$

$= \frac{1}{\frac{\sin^2 A + \cos^2 A}{\cos A . \sin A}}$

$= \frac{\cos A . \sin A}{\sin^2 A + \cos^2 A}$

$= \frac{\cos A . \sin A}{1}$

$= \cos A \sin A$

$\Rightarrow$ L.H.S. = R.H.S.

Hence $(\text{cosec } A - \sin A)(\sec A - \cos A) = \frac{1}{\tan A + \cot A}$ proved

(x) $\frac{1+\tan^2 A}{1+\cot^2 A} = \left(\frac{1-\tan A}{1+\cot A}\right)^2 = \tan^2 A$

Sol. L.H.S. $= \frac{1+\tan^2 A}{1+\cot^2 A}$

$= \frac{\sec^2 A}{\text{cosec}^2 A}$

$= \frac{\sin^2 A}{\cos^2 A}$

$= \tan^2 A$

Hence $\frac{1+\tan^2 A}{1+\cot^2 A} = \tan^2 A$ proved.

L.H.S. = $\left(\frac{1-\tan A}{1+\cot A}\right)^2$

$= \left(\frac{1-\frac{\sin A}{\cos A}}{1+\frac{\cos A}{\sin A}}\right)^2$

$= \left(\frac{\frac{\cos A-\sin A}{\cos A}}{\frac{\sin A-\cos A}{\sin A}}\right)^2$

$= \left(\frac{\cos A-\sin A}{\cos A} \times \frac{\sin A}{\sin A-\cos A}\right)^2$

$= \left(\frac{(\cos A-\sin A)}{\cos A} \times \frac{\sin A}{-(\cos A-\sin A)}\right)^2$

$= \left(\frac{-\sin A}{\cos A}\right)^2$

$= (-\tan A)^2$

$= \tan^2 A$

Hence $\frac{1+\tan^2 A}{1+\cot^2 A} = \left(\frac{1-\tan A}{1+\cot A}\right)^2 = \tan^2 A$ proved.